MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

SUPPLEMENTARY EXAMINATION

TRIMESTER 1, 2015/2016

EMT1016 – ENGINEERING MATHEMATICS I

(All Sections / All Groups)

18 NOV 2015
9.00 AM – 11.00 AM
(2 HOURS)

INSTRUCTIONS TO STUDENTS

1. This exam paper consists of **4 pages** (including cover page) with **4** Questions only.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of marks for each question is given.
3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Only NON-PROGRAMMABLE calculator is allowed.

Question 1

(a) Evaluate the following limit:

$$\lim_{x\to 2}\frac{\sqrt{2+x}-2}{x-2}$$ [4 marks]

(b) Given

$$f(x)=\begin{cases} k & x=0, \\ \dfrac{\tan(x)}{x} & x\in\left(\dfrac{-\pi}{2},0\right)\cup\left(0,\dfrac{\pi}{2}\right), \end{cases}$$

where k is a constant. Find k such that $f(x)$ is continuous at $x=0$.

[5 marks]

(c) Differentiate $y=2\theta\sqrt{\sin\theta}$ with respect to θ. [4 marks]

(d) Evaluate $\int_{e}^{e^2} x^3\ln(x)\,dx$. [6 marks]

(e) A region R is bounded by the curves $y=x^2-x-6$ and $y=x+2$. Sketch R and compute its area. [6 marks]

Continued...

Question 2

(a) If $u = \dfrac{p+q+r}{\sqrt{p^2+q^2+r^2}}$, find $2p\dfrac{\partial u}{\partial p} + 2q\dfrac{\partial u}{\partial q} + 2r\dfrac{\partial u}{\partial r}$ in its simplest form.

[8 marks]

(b) The electric power formula is given as $P = VI$, where $V = IR$. If $V = 100$ Volts and $R = 1\ k\Omega$, use total differential to estimate the change in P resulting from a decrease of 1 Volt in V and increase of 0.05 Ampere in I.

[7 marks]

(c) A cylindrical water tank with no cover has a surface area of 50π m^2 polyethylene, as illustrated in Figure Q2. Employ the method of Lagrange multipliers to compute the maximum possible volume of the tank. Show that the value obtained is indeed the maximum volume. [Hints: Volume of a cylinder, $V = \pi r^2 h$; Area of a topless cylinder, $A = 2\pi rh + \pi r^2$; where r = radius, h = height]

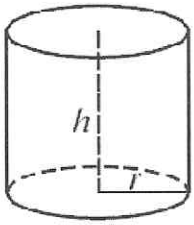

Figure Q2: Water tank

[10 marks]

Continued...

Question 3

(a) (i) Given that $\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n$ for $|x|<1$, find the Maclaurin series for $f(x)=\frac{1}{6+x}$. [6 marks]

(ii) Find the radius of convergence of the power series $\sum_{n=0}^{\infty}\frac{(2n)!}{(n!)^2}x^n$. [6 marks]

(b) Find all the four values of z that satisfy the equation $\left(\frac{z}{1-i}\right)^4=2+2i$. [13 marks]

Question 4

(a) A function $f(t)$ is defined over the finite interval $0\le t<2$ by

$$f(t)=2t-t^2.$$

(i) Show that the half-range Fourier sine series expansion of $f(t)$ is

$$\frac{32}{\pi^3}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^3}\sin\frac{(2k-1)\pi t}{2}.$$

[8 marks]

(ii) Sketch the graph of the periodic function corresponding to the Fourier series in part (i) from $t=-4$ to $t=6$. [4 marks]

(iii) To what value will the Fourier series in part (i) converge at the point $x=1$? Use this result to show that

$$\frac{\pi^3}{32}=1-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+\ldots.$$

[4 marks]

(b) Obtain the complex form of the Fourier series expansion of the following periodic function:

$$g(x)=4-\frac{2}{\pi}x,\ \text{for } 0\le x<2\pi,$$
$$g(x+2\pi)=g(x).$$

[9 marks]

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